Lidstone 边值问题正解的存在性

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摘 要: 通过选择合适的锥，并运用锥拉伸与压缩不动点定理，获得一类 Lidstone 型边值问题正解的存在性.

关键词: Listdone 型边值问题;　正解;　锥拉伸与压缩不动点定理.

Existence of positive solution of Lidstone boundary value problems

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Abstract: By choosing suitable cone ,and using fixed point theorem of cone expanssion and compression,we obtain the existence of positive solutions of a kind of Lidstone boundry value problems.

Keywords: Listdone boundary value problems; Positive solution; Fixed point theorem of cone expanssion and compression.

1　引言

本文讨论 Lidstone 型边值问题

$$(-1)^n u^{(2n)}(t) = f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)), \qquad 0 < t < 1, \tag{1.1}$$

$$u^{(2i)}(0) = u^{(2i)}(1) = 0, \qquad i = 0, 1, \cdots, n-1, \tag{1.2}$$

其中 $f : [0,1] \times \prod_{i=0}^{n-1}(-1)^i[0,+\infty) \longrightarrow (-1)^n[0,+\infty)$ 连续, 且 $n > 1$.

$2n$ 阶微分方程在力学和计算机科学与技术等领域中有很重要的应用. 例如, $n = 1$ 时, 文[1]利用临界点理论证明了二阶边值问题正解的存在性. $n = 2$ 时, (1.1), (1.2)可退化为四阶梁方程, 因此, 该类问题的研究得到了诸多学者的关注[7-12]. 在文[2]中, 作者通过运用 Guo-Krasnoselskii 不动点定理, 获得了四阶 Lidstone 边值问题解的存在性, 文[3]运用 Leggett-Williams 定理得到了四阶 Lidstone 边值问题对称解的存在性. 更一般地, 对于 $2n$ 阶 Lidstone 边值问题, 文[4]中, 运

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用 Krasnosel'skii 不动点定理在适当条件下获得 n 个正解的存在性, 文[5]运用 Rabinowitz 全局分歧定理获得 Lidstone 边值问题解的存在性, 文[6]讨论了多重正解的存在性, 但是其中大部分结果依赖于 f 在 $u=0$ 或 $u=\infty$ 处增长性限制. 那么, 当 f 仅满足某局部条件时, 是否也能得到正解的存在性结果? 为此, 本文通过构造合适的锥, 并运用锥拉伸与压缩的不动点定理, 试图获得问题(1.1),(1.2)正解的存在性.

本文的主要工具如下:

定理 1.1 设 E 是 Banach 空间,且 $K\subset E$ 是 E 中的一个锥. Ω_1, Ω_2 是 E 的开子集, $0\in\Omega_1$, $\overline{\Omega_1}\subset\Omega_2$, 若全连续算子

$$A: K\cap(\overline{\Omega_2}\setminus\Omega_1)\longrightarrow K$$

满足:

(1) $\|Au\|\le\|u\|$, $u\in K\cap\partial\Omega_1$, 并且 $\|Au\|\ge\|u\|$, $u\in K\cap\partial\Omega_2$ 或

(2) $\|Au\|\ge\|u\|$, $u\in K\cap\partial\Omega_1$, 并且 $\|Au\|\le\|u\|$, $u\in K\cap\partial\Omega_2$,

则 A 在 $K\cap(\overline{\Omega_2}\setminus\Omega_1)$ 上有一个不动点.

2 预备知识

引理 2.1 齐次边值问题

$$u^{(2n)}(t)=0,\qquad 0\le t\le 1$$

$$u^{(2i)}(0)=u^{(2i)}(t)=0,\qquad i=0,1,\ldots,n-1.$$

的格林函数 $G_n(t,s)$ 可表示为:

$$G_i(t,s)=\int_0^1 G(t,\tau)G_{i-1}(\tau,s)d\tau,\qquad i=2,3,\ldots,n,$$

其中, $G(t,s)$ 是二阶边值问题 $u''(t)=0,u(0)=u(1)=0$ 的格林函数,

$$G(t,s)=\begin{cases}t(1-s), & 0\le t\le s\le 1,\\ s(1-t), & 0\le s\le t\le 1.\end{cases}$$

进一步, $G_n(t,s)$ 满足:

$$\begin{cases}\dfrac{G_{n(t,s)}}{G_n(s,s)}\le 1, & t\in[0,1], s\in[0,1],\\[2mm] \dfrac{G_{n(t,s)}}{G_n(s,s)}\ge\dfrac{1}{4}, & t\in[\dfrac{1}{4},\dfrac{3}{4}], s\in[0,1].\end{cases}$$

$$\frac{G_{n(t,s)}}{G_{n-i}(s,s)}=\begin{cases}(\dfrac{1}{6})^i\dfrac{1-t}{1-s}\ge(\dfrac{1}{6})^i\cdot\dfrac{1}{4}, & 0\le s\le t\le\dfrac{3}{4},\ i=[1,n-1]_{\mathbb{Z}},\\[2mm] (\dfrac{1}{6})^i\dfrac{t}{s}\ge(\dfrac{1}{6})^i t\ge(\dfrac{1}{6})^i\cdot\dfrac{1}{4}, & \dfrac{1}{4}\le t\le s\le 1,\ i=[1,n-1]_{\mathbb{Z}}.\end{cases}$$

证明. 因为

$$G_1(t,s)=G(t,s)=\begin{cases}t(1-s), & 0\le t\le s\le 1,\\ s(1-t), & 0\le s\le t\le 1,\end{cases}$$

当 $0 \le t \le s \le 1$ 时,

$$G_2(t,s)=\int_0^1 t(1-\tau)\tau(1-s)d\tau=\frac{1}{6}t(1-s),$$

$$G_3(t,s)=\int_0^1 t(1-\tau)\frac{1}{6}\tau(1-s)d\tau=\frac{1}{6}G_2(t,s)=\frac{1}{36}t(1-s),$$

$$\vdots$$

$$G_n(t,s)=(\frac{1}{6})^{n-1}t(1-s).$$

同理, 当 $0 \le s \le t \le 1$ 时, $G_n(t,s)=(\frac{1}{6})^{n-1}s(1-t)$.

则有

$$\frac{G_{n(t,s)}}{G_n(s,s)} \le 1, \qquad t\in[0,1],$$

和

$$\frac{G_{n(t,s)}}{G_n(s,s)}=\begin{cases}\dfrac{1-t}{1-s}\ge\dfrac{1}{4}, & 0\le s\le t\le\dfrac{3}{4},\\ \dfrac{t}{s}\ge t\ge\dfrac{1}{4}, & \dfrac{1}{4}\le t\le s\le 1.\end{cases}$$

$$\frac{G_{n(t,s)}}{G_{n-i}(s,s)}=\begin{cases}(\dfrac{1}{6})^i\cdot\dfrac{1-t}{1-s}\ge(\dfrac{1}{6})^i\cdot\dfrac{1}{4}, & 0\le s\le t\le\dfrac{3}{4},\ i=[1,n-1]_{\mathbb{Z}}.\\ (\dfrac{1}{6})^i\cdot\dfrac{t}{s}\ge(\dfrac{1}{6})^i\cdot t\ge(\dfrac{1}{6})^i\cdot\dfrac{1}{4}, & \dfrac{1}{4}\le t\le s\le 1,\ i=[1,n-1]_{\mathbb{Z}}.\end{cases}$$

□

3 主要结论

设 $C[0,1]=\{u \mid u$ 在 $[0,1]$ 上连续$\}$, 其上范数为 $|u|_0=\max|u(t)|$. $C^{2(n-1)}[0,1]=\{u \mid u(t),u''(t),\cdots,u^{(2(n-1))}(t)\in C[0,1]\}$, 其上范数为 $\|u\|=\max\{|u|_0,|u''|_0,\cdots,|u^{(2(n-1))}|_0\}$. 定义非空集合 $K\subset C^{2(n-1)}[0,1]$ 为:

$$K=\{u\in C^{2(n-1)}[0,1]:u(t)\ge 0,(-1)^i u^{(2i)}(t)\ge\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\|u\|,\frac{1}{4}<t<\frac{3}{4},i=0,1,\ldots,n-1\}.$$

则 K 为 $C^{2(n-1)}[0,1]$ 中的一个锥.

定理 3.1 假设存在两个大于零的参数 λ 和 η 使得

(H_1) $(-1)^n f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\le\lambda(\int_0^1 G_n(t,s)ds)^{-1}$, $(t,u,u'',\cdots,u^{(2(n-1))})\in[0,1]\times\prod_{i=0}^{n-1}(-1)^i[0,\lambda]$,

和

(H_2) $(-1)^n f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\ge\eta(\int_{\frac{1}{4}}^{\frac{3}{4}}G_n(\frac{1}{2},s)ds)^{-1}$, $(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta,\eta]$.

成立. 则问题 (1.1),(1.2) 至少存在一个正解 u , 并且 $\|u\|$ 介于 λ 和 η 之间.

证明. 不失一般性, 我们假设 $\lambda<\eta$. 设 $u=u(t)$ 是(1.1), (1.2)的一个解, 则 u 是算子方程

$$u(t)=(-1)^n\int_0^1 G_n(t,s)f(s,u(s),u''(s),\ldots,u^{(2i)}(s),\ldots,u^{(2(n-1))}(s))ds:=Au(t)$$

的一个解.

当 $t \in [\frac{1}{4}, \frac{3}{4}]$ 时,

$$\begin{aligned}
Au(t) &= (-1)^n \int_0^1 G_n(t,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&= (-1)^n \int_0^1 \frac{G_n(t,s)}{G_n(s,s)} G_n(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&\geq \frac{1}{4}(-1)^n \int_0^1 G_n(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&= \frac{1}{4}|Au|_0
\end{aligned}$$

$$\begin{aligned}
Au(t) &= (-1)^n \int_0^1 G_n(t,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&= (-1)^n \int_0^1 \frac{G_n(t,s)}{G_{n-1}(s,s)} G_{n-1}(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds
\end{aligned}$$

$$\begin{aligned}
&\geq (-1)\frac{1}{4} \cdot \frac{1}{6}(-1)^{n-1} \int_0^1 G_{n-1}(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&= -\frac{1}{6} \cdot \frac{1}{4}|Au''|_0 \\
&\cdots
\end{aligned}$$

$$\begin{aligned}
Au(t) &\geq (-1)^i \cdot \frac{1}{4} \cdot (\frac{1}{6})^i (-1)^{n-i} \int_0^1 G_{n-i}(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&= (-1)^i \cdot \frac{1}{4} \cdot (\frac{1}{6})^i |Au^{(2i)}|_0 \\
&\cdots \\
Au(t) &\geq (-1)^{n-1} \frac{1}{4} \cdot (\frac{1}{6})^{n-1} |Au^{(2(n-1))}|_0.
\end{aligned}$$

则有 $Au(t) \geq \frac{1}{4} \cdot (\frac{1}{6})^{n-1} \|Au(t)\|$, $t \in [\frac{1}{4}, \frac{3}{4}]$.

同理, $(-1)Au''(t) \geq \frac{1}{4} \cdot (\frac{1}{6})^{n-2} \|Au(t)\|$, 且 $(-1)^i u^{(2i)}(t) \geq \frac{1}{4} \cdot (\frac{1}{6})^{n-1} \|u\|$.

设 $\Omega_1 := \{u \in K \mid \| u \| < \lambda\}$. 当 $u \in \partial\Omega_1$, 由 (H_1) 和引理 2.1 可得,

$$\begin{aligned}
Au(t) &= (-1)^n \int_0^1 G_n(t,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&\leq (-1)^n \int_0^1 G_n(s,s) f(s, u(s), u''(s), \ldots, u^{(2i)}(s), \ldots, u^{(2(n-1))}(s)) ds \\
&\leq \lambda (\int_0^1 G_n(s,s) ds)^{-1} (\int_0^1 G_n(s,s) ds) \frac{\| u \|}{\lambda} \\
&= \| u \| .
\end{aligned}$$

因此, $\| Au \| \leq \| u \|$, $u \in \partial\Omega_1$.

设 $\Omega_2 := \{u \in K \mid \| u \| < \eta\}$. 当 $u \in \partial\Omega_2$, 且由 K 的定义可得,

$$\begin{cases}
(-1)^i u^{(2i)}(t) \leq \| u \| = \eta, & t \in [0,1], \\
(-1)^i u^{(2i)}(t) \geq \frac{1}{4} \cdot (\frac{1}{6})^{n-1} \| u \| \geq \frac{1}{4} \cdot (\frac{1}{6})^{n-1} \eta, & t \in [\frac{1}{4}, \frac{3}{4}].
\end{cases}$$

有 $\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta\leq(-1)^i u^{(2i)}(t)\leq\eta, t\in[\frac{1}{4},\frac{3}{4}]$. 因此，由 (H_2) 可得,

$$\begin{aligned}Au(\frac{1}{2})&=(-1)^n\int_0^1 G_n(\frac{1}{2},s)f(s,u(s),u''(s),\ldots,u^{(2i)}(s),\ldots,u^{(2(n-1))}(s))ds.\\&\geq\eta(\int_{\frac{1}{4}}^{\frac{3}{4}}G_n(\frac{1}{2},s)ds)^{-1}(\int_{\frac{1}{4}}^{\frac{3}{4}}G_n(\frac{1}{2},s)ds)\frac{\parallel u\parallel}{\eta}\\&=\parallel u\parallel .\end{aligned}$$

因此，$\parallel Au\parallel\geq\parallel u\parallel$，$u\in\partial\Omega_2$.

综上, (1.1),(1.2) 至少存在一个正解 u , 且 $\lambda\leq\|u\|\leq\eta$.

□

注记 令

$$U=(u_0,u_1,\cdots,u_{n-1})\in R^n,\qquad |U|=\sqrt{u_0^2+u_1^2+\cdots+u_{n-1}^2}$$

且有

$$\max f_0:=\lim_{|U|\to 0^+}\max_{t\in[0,1]}\frac{f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))}{|U|},$$

$$\min f_0:=\lim_{|U|\to 0^+}\min_{t\in[0,1]}\frac{f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))}{|U|},$$

$$\max f_\infty:=\lim_{|U|\to\infty}\max_{t\in[0,1]}\frac{f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))}{|U|},$$

$$\min f_\infty:=\lim_{|U|\to\infty}\min_{t\in[0,1]}\frac{f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))}{|U|},$$

定理 3.2 令

$$(\int_0^1 G_n(t,s)ds)^{-1}=A,\qquad (\int_{\frac{1}{4}}^{\frac{3}{4}}G_n(\frac{1}{2},s)ds)^{-1}=B.$$

则有以下结果：

(a) 假设 $\max f_0:=C_1\in[0,A)$, 令$\epsilon=A-C_1>0$, 此时存在 $\lambda_1>0$ (λ_1 可选任意小) 使得

$$\max_{t\in[0,1]}\frac{f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))}{|U|}\leq\epsilon+C_1=A,$$

$(u,u'',\cdots,u^{(2(n-1))})\in\prod_{i=0}^{n-1}(-1)^i[0,\lambda_1]$. 则有,

$$f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\leq Au\leq A\lambda_1,$$

$(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[0,\lambda_1]$. 故定理 3.1 的假设((H_1))成立.

(b) 假设 $\min f_\infty := C_2 \in (\frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B, \infty)$, 令$\epsilon = C_2 - \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B > 0$, 此时存在 $\eta_1 > 0$ (η_1 可选任意大) 使得

$$\min_{t\in[0,1]} \frac{f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t))}{|U|} \geq -\epsilon + C_2 = \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B,$$

$(u, u'', \cdots, u^{(2(n-1))}) \in \prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_1, \eta_1]$. 则有,

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \geq \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}Bu \geq \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B\cdot\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_1 = B\eta_1,$$

$(t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_1, \eta_1]$, 故定理 3.1 的假设((H_2))成立.

(c) 假设 $\min f_0 := C_3 \in (\frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B, \infty)$, 令$\epsilon = C_3 - \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B > 0$, 此时存在 $\eta_2 > 0$ (η_2 可选任意小) 使得

$$\min_{t\in[0,1]} \frac{f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t))}{|U|} \geq -\epsilon + C_3 = \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B,$$

$(u, u'', \cdots, u^{(2(n-1))}) \in \prod_{i=0}^{n-1}(-1)^i(0, \eta_2]$. 则有,

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \geq \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}Bu \geq \frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B\cdot\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_2 = B\eta_2,$$

$(t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_2, \eta_2]$, 故定理 3.1 的假设 (H_2) 成立.

(d) 假设 $\max f_\infty := C_4 \in [0, A)$, 令 $\epsilon = A - C_4 > 0$, 此时存在 $\delta > 0$ (δ 可选任意大) 使得

$$(H_3)\ \max_{t\in[0,1]} \frac{f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t))}{|U|} \leq \epsilon + C_4 = A, (u, u'', \cdots, u^{(2(n-1))}) \in \prod_{i=0}^{n-1}(-1)^i[\delta, \infty)$$

成立. 则有以下两种情况:

(I) 假设 $\max_{t\in[0,1]} f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t))$ 是有界的, 即

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \leq L, \qquad (t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[0, \infty).$$

令 $\lambda_2 = \frac{L}{A}$ (L 可选任意大, λ_2 也可选任意大),

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \leq L = A\lambda_2,$$

$(t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i,\ [0, \lambda_2] \subseteq [0, 1] \times \prod_{i=0}^{n-1}(-1)^i[0, \infty)$.

(II) 假设 $\max_{t\in[0,1]} f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))$ 是无界的, 即存在 $\lambda_2\geq\delta$ (λ_2 可选任意大) 和 $t_0\in[0,1]$ 使得

$$f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\leq f(t_0,\lambda_2,\lambda_2,\cdots,\lambda_2),$$

$(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[0,\lambda_2]$. 由 $\lambda_2\geq\delta$ 和 (H_3) 可得

$$f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\leq f(t_0,\lambda_2,\lambda_2,\cdots,\lambda_2)\leq A\lambda_2,$$

$(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[0,\lambda_2]$.

由 (I) 和 (II) 成立, 可得定理 3.1 的假设 (H_1) 成立.

因此, 由定理 3.2 可有以下推论成立:

推论 1 假设

(1) $\max f_0:=C_1\in[0,A)$ 且 $\min f_\infty:=C_2\in(\frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B,\infty)$, 或

(2) $\min f_0:=C_3\in(\frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B,\infty)$ 且 $\max f_\infty:=C_4\in[0,A)$.

成立. 则问题 (1.1)-(1.2) 至少存在一个正解.

证明 由定理 3.1 和定理 3.2 可直接得到结果.

推论 2 假设

$(H_4)\min f_\infty:=C_2,\min f_0:=C_3\in(\frac{1}{\frac{1}{4}\cdot(\frac{1}{6})^{n-1}}B,\infty)$, 和

(H_5) 存在 $\lambda^*>0$, 使得 $f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\leq A\lambda^*$, $(t,u,u',\cdots,u^{2(n-1)})$ $\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[0,\lambda^*]$.

成立. 则问题 (1.1)-(1.2) 至少存在两个正解 u_1 和 u_1 满足 $0<\| u_1 \|<\lambda^*<\| u_2 \|$.

证明. 由定理 3.2,存在两个实数 η_1 和 η_2 满足 $0<\eta_2<\lambda^*<\eta_1$. 使得

$$f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\geq B\eta_1,$$

$(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_1,\eta_1]$, 和

$$f(t,u(t),u''(t),\cdots,u^{(2(n-1))}(t))\geq B\eta_2,$$

$(t,u,u'',\cdots,u^{(2(n-1))})\in[\frac{1}{4},\frac{3}{4}]\times\prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta_2,\eta_2]$, 成立.

则由定理 3.1 可得 (1.1)(1.2) 至少存在两个正解 u_1 和 u_1, 满足 $\eta_2<\| u_1 \|<\lambda^*<\| u_2 \|<\eta_1$.

推论 3 假设

$(H_6)\max f_0 := C_1, \max f_\infty := C_4 \in [0, A)$, 和

(H_7) 存在 $\eta^* > 0$, 使得 $f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \leq B\eta^*$, $(t, u, u'', \cdots, u^{(2(n-1))})$
$\in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[\frac{1}{4}\cdot(\frac{1}{6})^{n-1}\eta^*, \eta^*]$.

成立. 则问题 (1.1)(1.2) 至少存在两个正解 u_1 和 u_1 满足 $0 <\| u_1 \|< \eta^* <\| u_2 \|$.

证明 由定理 3.2, 存在两个实数 η_1 和 η_2 满足 $0 < \eta_1 < \eta^* < \eta_2$, 使得

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \leq A\lambda_1,\ (t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[0, \lambda_1],$$

和

$$f(t, u(t), u''(t), \cdots, u^{(2(n-1))}(t)) \leq A\lambda_2,\ (t, u, u'', \cdots, u^{(2(n-1))}) \in [\frac{1}{4}, \frac{3}{4}] \times \prod_{i=0}^{n-1}(-1)^i[0, \lambda_2]$$

成立. 则由定理 3.1 可得 (1.1)(1.2) 至少存在两个正解 u_1 和 u_1 满足

$$\eta_1 <\| u_1 \|< \eta^* <\| u_2 \|< \eta_2.$$

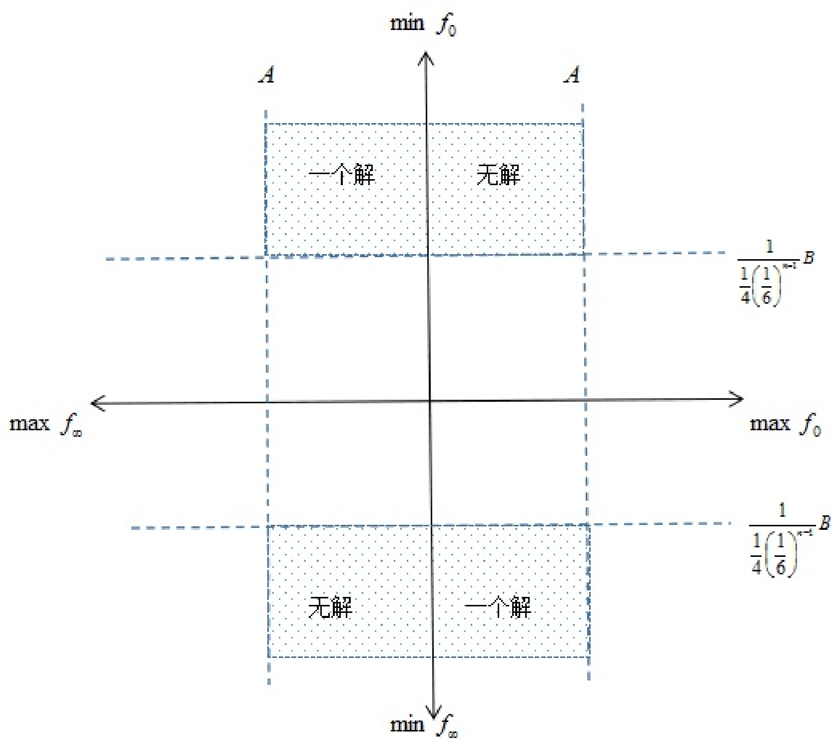

FIGURE 2.1

由推论1,2,3可得Figure2.1.

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